Effect as a pivot between actions and symbols: the case of vector

by

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Declaration

I hereby certify that this thesis is my own work. Portions of the Preliminary Study chapter (Chapter 4) were used in two papers: one submitted to the 26th Annual Conference of the International Group for the Psychology of Mathematics Education (PME) in Norwich, UK, in 2002; a second one submitted to the Mediterranean Journal of Mathematical Education in Cyprus, in 2003. The references to the papers are included in the References of this thesis. I also confirm that this thesis has not been submitted for a degree at any other university.

Abstract

Students' difficulties with vectors in Mechanics, at AS and A level, have been considered in a number of studies to date. Some of the research considers how students' intuitions arise from working in different contexts and how it affects their problem-solving capabilities, others think that pre-Newtonian views affect students' thinking. There are considerations that the idea of vector has different meanings in different contexts and therefore it is not easily transferable from one context to another. There are suggestions in the literature that a qualitative approach to teaching would help students to learn. None of the research studied considers the important idea of focusing on the vector concepts that are *common* to the various contexts, instead they are more concerned with the problems caused by the *differences* between them. Nor do they focus on the compression of a vector as an action into the more flexible idea of a free vector as a single mental object that can be represented by any arrow of given magnitude and direction.

In this thesis an approach is developed to base the students' experience on manipulating physical objects, to focus on the *effect* of a translation rather than on the action itself. The essential idea is to notice that every point on the object translated (and on the hand doing the translation) moves in the same direction and travels the same distance. The *effect* of the translation is therefore represented by *any* arrow of this magnitude and direction, leading to the notion of free vector. From the same viewpoint, the sum of two vectors is simply the single translation having the same effect as the combination of one translation followed by the other.

The main hypothesis is that:

> Teachers can help students develop the notion of a translation as a free vector through focusing on the effects of physical actions, linking graphic and symbolic representations, so that the concept of free vector is constructed as a cognitive unit that may be used in a versatile way in a range of different contexts.

This was tested by a comparative study of two classes using both quantitative and qualitative methods. The control group carried on with the normal programme of study while the experimental group was exposed to lessons focusing on of the notion of a free vector as the effect of a transformation. The students' own constructions were supported by activities and discussions in reflective plenary sessions. The results of the study revealed that there were significant changes in the students following the experimental programme, in which they were more likely to conceive of the symbols for vectors as cognitive units operating in a flexible and versatile manner. The quantitative improvement was sustained and increased over a longer period.

Interviews with the teachers revealed differences between the mathematics and physics teachers' perceptions of the students' expected difficulties. Interviews with the students revealed the more successful interviewees referring to the concept of vector as a single cognitive unit across contexts, while the less successful tended to consider the concept of vector operating in different ways as journeys and as forces.

Both quantitative and qualitative data show significant conceptual changes in students following the experimental approach; these changes were more marked over the longer period of time between pre-test and delayed post-test.